Regular Expressions

Lecture 11
Section 3.2

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Outline

1. Equivalence of Regular Expressions and DFAs
2. Assignment

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2. Assignment

Regular Languages and Regular Expressions

Theorem

A language is regular if and only if it is the language of some regular expression.

Regular Languages and Regular Expressions

Proof ($\Leftarrow$).

- The basic languages $L(a)$, $L(\lambda)$, and $L(\varnothing)$ are regular.
- Union, concatenation, and Kleene star of regular languages are regular.
- Therefore, the language of a regular expression is regular.

$\square$

Proof ($\rightarrow$), beginning.

- Let L be a regular language.
- We need to construct a regular expression r such that $L(r) = L$.
- We will use a generalized transition graph (GTG).

$\square$

Definition

A generalized transition graph is like a regular transition graph except that the labels are regular expressions. To make a transition from one state to another, we must read a *string* that matches the regular expression labeling that transition.

Converting a DFA to a Regular Expression

Proof, continued.

- Begin with a transition diagram for L.
- Replace each label (symbol) with the equivalent regular expression.
- Add a new start state that has no transitions into it, but has one λ-move from it to the original start state.
- Add a new accept state that has no transitions out of it, but has λ-moves into it from all of the original accept states.
- Remove all states from which the accept state is inaccessible.

□

Converting a DFA to a Regular Expression

Proof, continued.

- Note that the number of states in the GTG is at least 3.
- The proof will reduce the number of states down to 2, at which point we will have the regular expression.

□

Converting a DFA to a Regular Expression

Proof, continued.

- Choose a non-initial, non-final state q to be removed.
- For every state p with a transition into q and for every state r with a transition from q, create a transition from p to r, as follows.
- Let
  - r_1 be the label on the transition $p \to q$.
  - r_2 be the label on a loop $q \to q$, if there is one.
  - r_3 be the label on the transition from $q \to r$.
- Apply the label $r_1 r_2^* r_3$ to the new transition.

□

Converting a DFA to a Regular Expression

Proof, concluded.

- After doing this for every combination of transitions into q and out of q, remove state q and all of its transitions.
- Repeat this process until only the initial and final states remain.
- The label on that single remaining transition is the regular expression.

□

Example

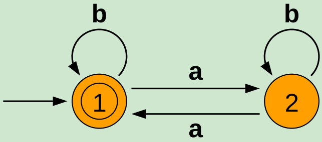
Example (Converting a DFA to a regular expression)

- Find a regular expression for the language

$$L = \{w \mid w \text{ has an even number of } \mathbf{a}\text{'s}\}.$$

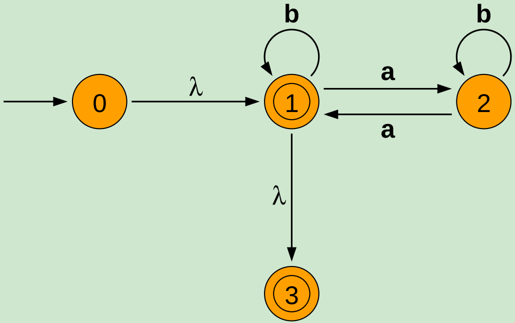
Example

Example (Converting a DFA to a regular expression)



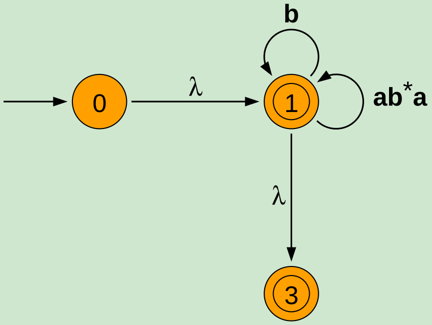
Example

Example (Converting a DFA to a regular expression)



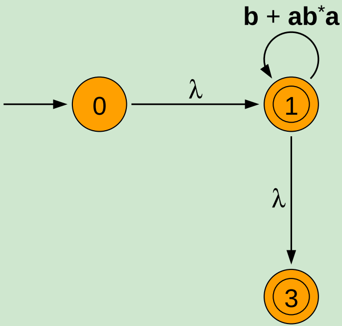
Example

Example (Converting a DFA to a regular expression)



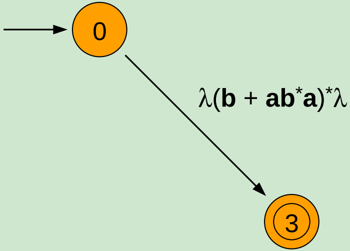
Example

Example (Converting a DFA to a regular expression)



Example

Example (Converting a DFA to a regular expression)



Example

Example (Converting a DFA to a regular expression)

- Therefore,

$$L((\mathbf{b} + \mathbf{ab}^*\mathbf{a})^*) = \{w \mid w \text{ has an even number of } \mathbf{a}\text{'s}\}.$$

- This regular expression "parses" the string

$$\mathbf{babbaababbbaab}$$

as

$$\mathbf{b|abba|aba|b|b|b|aa|b.}$$

Example

Example (Converting a DFA to a regular expression)

- Find regular expressions for the following languages
  - All strings containing an odd number of **a**'s.
  - All strings containing an even number of **a**'s and an even number of **b**'s.
  - All strings that do not contain **aaa**.

Collected

To be collected on Mon, Sep 19:

- Section 2.2 Exercises 14, 23.
- Section 2.3 Exercises 5, 16.
- Section 2.4 The exercise at the end of Lecture 9.

Outline

Assignment

Assignment

- p. 90: 1, 7, 10, 12b, 15ac.